Predicting Recidivism in the context of Mental Health Status: A Markov Chain Approach

Math 22A Final Project
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0 Cover Letter

Over the course of this project, Karen and Rhea collaborated heavily on most sections, although each person focused their efforts on different sections. Rhea introduced the mathematical concepts, wrote the steady-state vector example, and created the transition matrix used in the model, while Karen wrote the other example and proved the key theorem. Both of them collaborated on researching the topic, integrating sociological concepts, and analyzing the data to produce and interpret our models.

Following peer reviews, we incorporated several suggestions. For example, many students commented on the lack of examples surrounding steady-state vectors, so we modified our examples to introduce this topic on a computational level. We also cut one example out of our paper to eliminate observed redundancies. Additionally, we changed some LaTeX formatting, such as including boxes around theorems, to increase paper readability as per the comments. We also increased the amount we expressed ideas through mathematical notation, rather than simply through words.

Our meetings with our assigned project advisor, Caleb, were particularly helpful. At our first meeting before the project draft deadline, we discussed with him how the lack of data from Massachusetts hindered our ability to build a cohesive Markov chain model, so we switched to Florida following his suggestion.

In our second meeting with Caleb to go over our draft, we discussed how to more clearly present information in our paper (e.g. explicitly marking out key definitions). He also gave us many helpful suggestions on how to go about proving the uniqueness aspect of the proof we discuss in our paper.

With the help of our classmates and Caleb, we were able to produce this final draft which we hope gives the reader a comprehensive understanding of Markov chains and their particular application to predicting recidivism.

1 Introduction

1.1 Background

Sociology is the study of human behaviour within the context of social relationships, institutions, and systems. Through this project, we wanted to study how linear algebra can be applied to one area of focus in sociology: the prison system, specifically how to determine the chance of recidivism in a given population. **Recidivism** refers to the percent chance that someone who committed an offense will re-offend. In particular, we are interested in how mental health can affect these recidivism rates.

In a survey of inmates conducted by the US Bureau of Justice Statistics, an estimated 56% of state prisoners and 64% of jail inmates reported symptoms of mental health disorders or had a recent history of mental health problems. Repeated encounters with the criminal justice system are also more common among people with a serious mental illness. Among inmates in prison or jail who had a mental health problem, approximately 25% had served three or more prior incarcerations, compared to around 20% among those without mental health problems [7].

Given that the U.S. has the largest prison population in the world and its prison system has been and continues to be the subject of controversy and debate, we decided to study this issue through the quantitative lens offered by linear algebra techniques. Overall, we hope our project can inform a greater understanding of the importance of adequate and continued mental health treatments for offenders, both incarcerated and released, and ultimately highlight an area of focus for further policy evaluation on the state and federal level.

1.2 Introduction to Markov Chains

In order to study recidivism, Markov chains are used to model sequences of events where the probability of some outcome occurring in a particular event is dependent on the outcome that occurred in the event directly preceding it.

1.2.1 Important Definitions

Definition 1 ***Probability vectors*** *are vectors of the form* $< t_1, ..., t_n >$ *where each element is non-negative and where* $t_1 + ... + t_n = 1$.

Definition 2 ***Stochastic matrices*** *are* $n \times n$ *matrices with probability vectors, each with* n *elements, as columns.*

Definition 3 *Given a series of probability vectors,* $\boldsymbol{x_0}$, $\boldsymbol{x_1}$, $\boldsymbol{x_2}$... *all in* $\mathbb{R}^n$, *and a stochastic* $n \times n$ *matrix* P, *the resultant* ***Markov Chain*** *is*

$$\boldsymbol{x_1} = P\boldsymbol{x_0},\ \boldsymbol{x_2} = P\boldsymbol{x_1},\ \cdots$$

which can also be represented more generally as

$$\boldsymbol{x_{k+1}} = P\boldsymbol{x_k}$$

for all k in the set of whole numbers.

Definition 4 *A **steady state vector $\mathbf{q}$** is a probability vector such that $A\mathbf{q} = \mathbf{q}$, i.e. an eigenvector of stochastic matrix A associated to the eigenvalue 1.*

Definition 5 *A **regular stochastic matrix** is a nonnegative stochastic matrix such that all the entries of some power of the matrix are positive, i.e. for a matrix P, there exists some power $k \geq 1$ such that each entry of P^k is nonzero.*

1.2.2 Analyzing systems with Markov Chains

Each element in the **probability vector** represents the probability of each of n possible outcomes occurring at the given point in the sequence of events. Many times, the probability vectors are called **state vectors** as they show a probabilistic distribution of states that a system could be in at a given point. Markov chains can be useful to predict what will happen in the short term, but they are especially useful at providing a mathematical framework to model long-term trends in a wide range of fields.

Often when looking at distant state vectors, we see that they steadily approach a certain vector, known as the **steady state** or equilibrium vector for the given **stochastic matrix** P. Once this vector has been reached, then multiplication by P still results in the same vector, so the probability of outcomes occurring becomes constant from one state to the next.

1.3 Computational examples relating to Markov Chains

1.3.1 Example 1: Representing Population Movement[1]

Let M be a migration matrix representing population movement between Cambridge and Boston.

Let's say that 5% of the Cambridge population moves to Boston annually, while 95% stays in Cambridge. Thus, the first column of M, representing the geographic distribution next year of the current Cambridge population, is $\begin{bmatrix} 0.95 \\ 0.05 \end{bmatrix}$. On the other hand, let's say 4% of the Boston population moves to Cambridge, while 96% stays in Boston annually, so the second column of M is $\begin{bmatrix} 0.04 \\ 0.96 \end{bmatrix}$.

Putting this together,

$$M = \begin{bmatrix} 0.95 & 0.04 \\ 0.05 & 0.96 \end{bmatrix}$$

That is, annually 5% of the Cambridge population moves to Boston, and 4% of the Boston population moves to Cambridge. M is a stochastic matrix since its columns are probability vectors.

[1] Adapted from Lay, Linear Algebra[8]

In 2019, the population of Cambridge was 116,632 and the population of Boston was 684,379 (US Census Bureau).

Let $\text{pop}_0 = \begin{bmatrix} 116,632 \\ 683,379 \end{bmatrix}$. To predict the populations of each city in 2020, we would calculate $M(\text{pop}_0)$ to get:

$$\text{pop}_1 = \begin{bmatrix} 138135.56 \\ 661875.44 \end{bmatrix}$$

1.3.2 Example 2. Steady State Vectors[2]

Let's say that if you go to the gym on one day, then there's a 90% chance that you will go to the gym the next day. However, if you don't go to the gym, then there is only a 60% chance that you will go to the gym the next day due to lack of motivation.

Thus,

$$P = \begin{bmatrix} 0.9 & 0.6 \\ 0.1 & 0.4 \end{bmatrix}$$

Assuming you go to the gym on the first day (day 0), let x_n be the probability that you go to the gym on the nth day after that.

Then, $x_1 = \begin{bmatrix} 0.9 & 0.6 \\ 0.1 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 0.9 & 0.6 \\ 0.1 & 0.4 \end{bmatrix} x_1 = \begin{bmatrix} 0.87 \\ 0.13 \end{bmatrix}$

Continuing this process, we find that

$$x_3 = \begin{bmatrix} 0.861 \\ 0.139 \end{bmatrix}, x_4 = \begin{bmatrix} 0.8583 \\ 0.1417 \end{bmatrix}, x_5 = \begin{bmatrix} 0.8575 \\ 0.1425 \end{bmatrix}$$

We can see that the values tend to be approaching the same values, and the steady-state vector, denoted as x_s, is the vector that is comprised of the values being approached.

Recall that steady-state vectors make the following equation true:

$$x_s = P * x_s$$

Thus, letting I be the 2×2 identity matrix, $x_s - P * x_s = (I - P)x_s = 0$

Let us further define x_s as being equal to $\begin{bmatrix} a \\ b \end{bmatrix}$

Thus, $(I - P)x_s = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.9 & 0.6 \\ 0.1 & 0.4 \end{bmatrix} \right) \begin{bmatrix} a \\ b \end{bmatrix} = \left(\begin{bmatrix} 0.1 & -0.1 \\ -0.6 & 0.6 \end{bmatrix} \right) \begin{bmatrix} a \\ b \end{bmatrix} = 0$

[2] Adapted from Bloomington Tutors, bloomingtontutors.com/blog/going-steady-state-with-markov-processes

After completing matrix multiplication, we obtain the following equation:

$$0.1a - 0.6b = 0$$

Recall that by definition of a probability vector, $a + b = 1$

We express this system of equations with the following augmented matrix:

$$\left[\begin{array}{cc|c} 0.1 & -0.6 & 0 \\ 1 & 1 & 1 \end{array}\right]$$

Through row reduction, we obtain:

$$\left[\begin{array}{cc|c} 1 & 0 & 6/7 \\ 0 & 1 & 1/7 \end{array}\right]$$

Now, we see that our steady-state vector is $x_s = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 6/7 \\ 1/7 \end{bmatrix}$

So, as more days pass, the chance that you will go to the gym on some day in the future approaches 6/7 while the chance that you won't approaches 1/7.

1.4 Proof of critical Markov Chain theorems

We first introduce the **Perron-Frobenius Theorem**, which we will use to prove another theorem, **Theorem 1.2**, that focuses on the existence and uniqueness of steady state vectors of Markov chains with regular stochastic transition matrices. We will then prove that with such transition matrices, the Markov chain converges to the steady state vector as it approaches infinity.

Theorem 1.1 ***(Perron-Frobenius)*** *For any strictly positive matrix $A > 0$ there exists $\lambda_0 > 0$ and $x_0 > 0$ such that*

- $Ax_0 = \lambda_0 * x_0$;
- *if $\lambda \neq \lambda_0$ is any other eigenvalue of A, then $|\lambda| < \lambda_0$;*
- *λ_0 has geometric and algebraic multiplicity 1, i.e. the dimension of the 1-eigenspace is 1 and the number of times 1 appears as a root of the characteristic polynomial is 1.*

Corollary 1.1.1 *The 1-eigenspace of a positive stochastic matrix is a line.*

Corollary 1.1.2 *The 1-eigenspace contains a vector with positive entries.*

Corollary 1.1.3 *All vectors approach the 1-eigenspace upon repeated multiplication by A.*

Corollary 1.1.4 *If A is a regular matrix, then the conclusions of Theorem 1.1 hold also for A.*

Theorem 1.2 *If P is an $n \times n$ regular stochastic matrix, then P has a unique steady-state vector q. Further if x_0 is any initial state and $x_{k+1} = Px_k$ for k = 0,1, 2, ... , then the Markov chain x_k converges to q as k approaches infinity.*

Let us show that every regular stochastic matrix has a steady-state vector, and that it is unique.

Proof of Existence and Uniqueness of Steady State Vector:
If P is stochastic, then each row of P^T sums to 1. Multiplying P^T by the $n \times 1$ column vector of 1's returns the sums of each row, i.e. the n x 1 column vector of 1's.

$$P^T * \mathbf{1} = 1 * \mathbf{1}$$

Therefore, 1 is an eigenvalue of P^T, and the eigenvector is a column vector of 1's. Note that P and P^T have the same eigenvalues as they have the same characteristic equations:

$$det(P - \lambda I_n) = det((P - \lambda I_n)^T) = det(P^T - \lambda I_n).$$

Therefore, 1 is also an eigenvalue of P, so there exists a corresponding eigenvector $\boldsymbol{q}$ such that

$$P * \boldsymbol{q} = 1 * \boldsymbol{q}$$

By Corollary 1.1.4, P has a unique largest eigenvalue $\lambda \in R$. Since the geometric multiplicity of λ (1) is 1, $\mathbf{q}$ is a unique stochastic vector such that $P * \boldsymbol{q} = \boldsymbol{q}$ ■.

Lemma 1.3 ***(Jordan Normal Form)*** *Let $A \in \mathbb{C}^{n \times n}$ be any matrix with eigenvalues $\lambda_1, \cdots, \lambda_l \in \mathbb{C}$, $l \leq n$. Then there exists an invertible matrix $U \in \mathbb{C}^{n \times n}$ such that*

$$UAU^{-1} = \begin{bmatrix} J_1 & 0 & \cdots & 0 \\ 0 & J_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & J_r \end{bmatrix}$$

where each J_i is a $k_i \times k_i$ ***Jordan block*** *associated to some eigenvalue λ of A:*

$$J_i = \begin{bmatrix} \lambda & 1 & 0 & \cdots & 0 & 0 \\ 0 & \lambda & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda & 1 \\ 0 & 0 & 0 & \cdots & 0 & \lambda \end{bmatrix}$$

The total number of blocks associated to a given eigenvalue λ corresponds to λ's geometric multiplicity, and their total dimension $\Sigma_i k_i$ to λ's algebraic multiplicity.

Proof of Convergence of Regular Markov Chains:
Let us consider the Jordan normal form of transition matrix P, where P is a regular stochastic matrix. For simplicity, we assume that all eigenvalues of P, $\lambda_1, \cdots, \lambda_n$ are real and distinct. Then, the rows of U may be taken to be the right eigenvectors of P, and the Jordan normal form reduces to the following eigenvalue decomposition:

$$UPU^{-1} = \Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}$$

Note that the columns of $U^-1 = V$ are the left eigenvectors corresponding to eigenvalues $\lambda_1, \cdots, \lambda_n$. As proven above, P has a unique largest eigenvalue $\lambda = 1$, and other eigenvalues may be ordered so that $1 > |\lambda_2| \leq \cdots \leq |\lambda_l|$. The unique right eigenvector associated with $\lambda_1 = 1$ is the steady state vector $\mathbf{q}$, and the corresponding unique left eigenvector is $\mathbf{1}$. Normalizing the first row of $V = U^{-1}$,

$$\Lambda = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}$$

Thus, we have

$$P^2 = (V\Lambda U)^2 = V\Lambda^2 U = V \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \lambda_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n^2 \end{bmatrix} U$$

and in general

$$P^k = V\Lambda^k U = V \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \lambda_2^k & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n^k \end{bmatrix} U \xrightarrow{k \to \infty} V \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix} U = \begin{bmatrix} v_{11}u_1 \\ v_{12}u_1 \\ \vdots \\ v_{1n}u_1 \end{bmatrix} = \begin{bmatrix} q \\ q \\ \vdots \\ q \end{bmatrix}$$

Therefore, the Markov chain of P converges to $\mathbf{q}$ as k approaches infinity. ■

2 Application to Recidivism

Lin, Muser, Munsell, Benson, Menzin (2014) used a Markov state transition model to estimate the number of schizophrenia patients recently released from jail who would experience psychiatric relapse and/or arrest and reincarceration over a 3-year period. A Markov model was chosen for their analysis given that modeling criminal justice costs and outcomes requires following individuals and evaluating their transitions between discrete states: incarceration, in the community, etc. In their model, three healths states were considered: (in community, on therapy), (in community, off therapy), (incarcerated) [1].

We followed a similar model framework as Lin et al. (2014), but focusing not on economic costs of psychiatric treatment and rather on predicting recidivism numbers of inmates with regards to the mental health status at release.

3 Analysis of Florida Dataset

3.1 Dataset

We got the following data (Figure 1) from the Florida Department of Corrections 2020-2021 Recidivism Report [5]. We used this data to create a Markov chain model to study future recidivism rates and how changing mental health treatment can affect them.

	Male				Female			
	Total Returned	Total Released	Recidivism Rate	Percent in the Male Category	Total Returned	Total Released	Recidivism Rate	Percent in the Female Category
Mental Health Status at Release								
Inmate does not require on-going treatment	5,121	20,719	24.7%	86.7%	237	2,026	11.7%	59.3%
Inmate Requires on-going treatment	1,016	3,176	32.0%	13.3%	199	1,392	14.3%	40.7%
Total	6,137	23,895	25.7%	100.0%	436	3,418	12.8%	100.0%
Substance Abuse Treatment Needed								
No	1,845	7,521	24.5%	31.5%	76	767	9.9%	22.4%
Yes	4,292	16,374	26.2%	68.5%	360	2,651	13.6%	77.6%
Total	6,137	23,895	25.7%	100.0%	436	3,418	12.8%	100.0%
Primary Offense								
Murder/Manslaughter	102	545	18.7%	2.3%	6	76	7.9%	2.2%
Sexual/Lewd behavior	346	1,330	26.0%	5.6%	6	14	42.9%	0.4%
Robbery	530	1,799	29.5%	7.5%	25	166	15.1%	4.9%
Violent, Other	985	3,715	26.5%	15.5%	63	501	12.6%	14.7%
Burglary	1,197	4,116	29.1%	17.2%	53	357	14.8%	10.4%
Property Theft/Fraud/ Damage	975	3,591	27.2%	15.0%	123	936	13.1%	27.4%
Drugs	1,109	5,257	21.1%	22.0%	124	1,089	11.4%	31.9%
Weapons	346	1,542	22.4%	6.5%	4	72	5.6%	2.1%
Other	539	1,974	27.3%	8.3%	31	197	15.7%	5.8%
Total (excluding missing)	6,137	23,895	25.7%	100.0%	436	3,418	12.8%	100.0%

Figure 1: Recidivism Rates Table (2020-21, Florida)

3.2 Markov Model Schematic

In order to construct a transition matrix, we first need to determine the states a person in the population could be in. Assuming the base population comprises of individuals released from incarceration, we have the following schematic to represent the transition of states from this point onward for them:

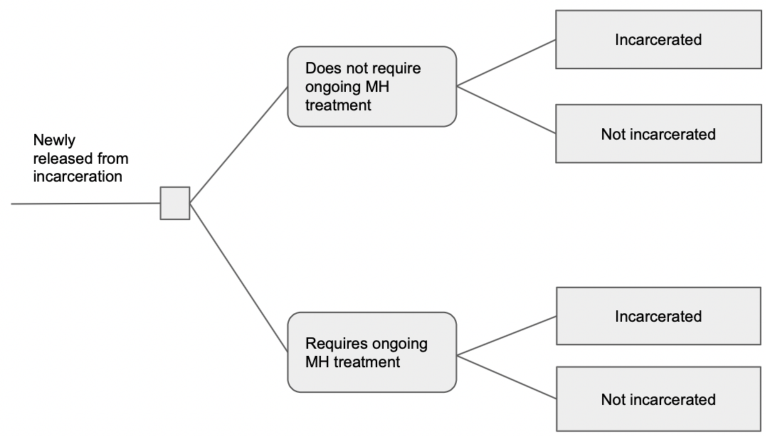

Figure 2: Recidivism Rates Table Pt. 2 (2020-21, Florida)

Thus, there are four different states that a person can be in: in jail requiring mental health treatment, in jail without requiring mental health treatment, in the community requiring mental health treatment, and lastly in the community without requiring mental health treatment.

In order to form our transition matrix P for this situation, we must find four column vectors that represent the likelihood that a person will be in each of these four stages at the next step given that they are in a specific one of these stages currently.

3.2.1 Model Assumptions

In order to construct the transition matrix P we incorporated information from the Florida data set as well as other research. Van den Berg et al. (2016) studied the changes in depression and stress of inmates after release from tobacco-free prisons in the United States. They found that although most inmates improved after prison, 30.8% had a worsening in levels of depression between baseline and the three-week follow-up. 29.8% had a worsening in levels of stress after release than during incarceration [2]. Therefore, we assumed a 30% chance that a member of our recently released population would develop a mental illness in the community.

We assumed that the release rate between those who needed ongoing mental health treatment and those who did not would be the same, and we calculated this shared release rate through looking at how many inmates were released in 2017 and then dividing that value by how many were admitted that year in addition to the initial population of inmates in Florida in 2017, before arriving at a 24.4% release rate [7]. For transitioning from the community to jail, we used a weighted average between males and females of the recidivism rates provided in the data table, finding a 22.46% recidivism rate for those who do not require mental health treatment versus a 28.51% rate for those who do. [5]

Lastly, we assumed that for those already in jail, their mental health status would remain constant while they remain in jail, and we assumed that becoming incarcerated would not eliminate the need for mental health treatment for someone in the community who required treatment immediately prior to becoming incarcerated.

3.2.2 Using Data to Form Transition Matrix

Through this we obtained the following completed transition matrix for our Markov chain.

- J, MH = In jail, requiring mental health treatment
- J, NMH = In jail, not requiring mental health treatment
- C, MH = In the community, requiring mental health treatment
- C, NMH = In the community, not requiring mental health treatment

$$\mathbf{P} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} \text{J, MH} & \text{J, NMH} & \text{C, MH} & \text{C, NMH} \\ \end{array}$$

$$\mathbf{P} = \begin{bmatrix} .756 & 0 & .285 & .101 \\ 0 & .756 & 0 & .123 \\ .244 & 0 & .715(1-m) & .233 \\ 0 & .244 & .715(m) & .543 \end{bmatrix} \begin{array}{l} \text{J, MH} \\ \text{J, NMH} \\ \text{C, MH} \\ \text{C, NMH} \end{array}$$

Note: m represents the percentage of released inmates who require mental health treatment in their current state, but in 3 years no longer require treatment. We set this as a variable in order to measure the effects of increased treatment post-release on the mental health status of the inmates concerned.

Initial Population (x_0): In FY 2017-2018, 20,719 males and 2,026 females were released who did not require on-going treatment, totalling 22,745 (see Figure 2). A total of 4568 inmates required on-going treatment at release. For those returning to jail, 5,358 did not require on-going treatment and 1,215 did require on-going treatment.

$$\text{pop}_0 = \begin{bmatrix} 1,215 \\ 5,358 \\ 4,568 \\ 22,745 \end{bmatrix}$$

3.3 Deriving Steady State Vectors of Markov Chain

Case 1: m = 0.40 *In other words, the effectiveness of continued mental health treatment (to the point where treatment is no longer required) for inmates released to the community is 40%.*

Let us define our steady-state vector, denoted as x_s, as the vector that satisfies the following equation:

$$x_s = P_{0.4} * x_s.$$

Moving things around, this is equivalent to $(I - P_{0.4})x_s = 0$. Let us further define x_s as being equal to $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$.

$$\text{Thus, } (I - P_{0.4})x_s = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .756 & 0 & .285 & .101 \\ 0 & .756 & 0 & .123 \\ .244 & 0 & .429 & .233 \\ 0 & .244 & .286 & .543 \end{bmatrix} \right) x_s$$

$$= \begin{bmatrix} .244 & 0 & -.285 & -.101 \\ 0 & .244 & 0 & -.123 \\ -.244 & 0 & .571 & -.233 \\ 0 & -.244 & -.286 & .457 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$

$$\text{rref}(I - P_{0.4}) = \begin{bmatrix} 1 & 0 & 0 & -1.778 \\ 0 & 1 & 0 & -.504 \\ 0 & 0 & 1 & -1.168 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using this, we determine $a = 1.778d, b = .504d$, and $c = 1.168d$. Recall as well that by definition of a probability vector, $a + b + c + d = 1$. We can substitute these values into this last equation to obtain:

$$1.778d + .504d + 1.168d + d = 4.45d = 1$$

So, $d = .225$, and consequently, $a = .400$, $b = .113$, and $c = .262$. The steady state vector of transition matrix $P_{0.4}$ is

$$x_s = \begin{bmatrix} 0.400 \\ 0.113 \\ 0.262 \\ 0.225 \end{bmatrix}.$$

Multiplying by our initial population count of 33,886, we get

$$\text{pop}_s = \begin{bmatrix} 13555 \\ 3829 \\ 8878 \\ 7624 \end{bmatrix}$$

Case 2: m = 0.60 *In other words, the effectiveness of continued mental health treatment (to the point where treatment is no longer required) for inmates released to the community is 60%.*

Similar to Case 1, we must find the steady state vector x_s such that $(I - P)x_s = 0$.

Following the same process as Case 1,

$$\text{rref}(I - P_{0.6}) = \begin{bmatrix} 1 & 0 & 0 & -1.323 \\ 0 & 1 & 0 & -.504 \\ 0 & 0 & 1 & -.779 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So, after creating a system of equations and solving that system, we obtain the following $x_s = \begin{bmatrix} 0.367 \\ 0.140 \\ 0.216 \\ 0.277 \end{bmatrix}$. Thus, $\text{pop}_s = \begin{bmatrix} 12436 \\ 4744 \\ 7319 \\ 9386 \end{bmatrix}$

3.4 Predicting Recidivism Numbers

We can also use this Markov Chain model to predict numbers in the short-term. For example, assuming $m = 0.4$ for calculation purposes, we can find $\text{pop}_{1,\ 0.4}$, or the distribution of the population across the four states in 2018, through matrix multiplication.

$$\text{pop}_{1,\ 0.4} = P_{0.4}(\text{pop}_0) = \begin{bmatrix} 4,518 \\ 6,848 \\ 7,556 \\ 14,964 \end{bmatrix}.$$

In 10 years, the population will look like

$$\text{pop}_{10,\ 0.4} = P_{0.4}^{10}(\text{pop}_0) = \begin{bmatrix} 12,617 \\ 4,674 \\ 8,564 \\ 8,030 \end{bmatrix}.$$

Likewise, for $m = 0.6$, the prison and community population will look like

$$\text{pop}_{10,\ 0.6} = P_{0.6}^{10}(\text{pop}_0) = \begin{bmatrix} 11,753 \\ 5,358 \\ 7,115 \\ 9,659 \end{bmatrix}.$$

4 Implications of Analysis

4.1 Discussion of Results

In the first case, where the effectiveness of mental health treatment is $m = 40\%$, we found that the chance of inmates entering the community not requiring mental health treatment is lower than the case where effectiveness is $m = 60\%$ Out of our initial population of around 34,000, approximately 2,000 more of them were in the community long-term without needing ongoing mental health treatment, which is the ideal state out of the four. Especially when scaled up beyond the state of Florida, these findings show that increasing the effectiveness of mental health treatments in the community can have significant positive impacts.

4.2 Modifications to Markov chain models to improve accuracy

We would like to consider a more robust model that takes into account more factors that affect recidivism, such as age at discharge, geographic environment (e.g. county within a state), and criminal history (repeat vs. first-time offenders). Also, due to lack of available evidence, some of the percentages in our transition matrix are estimates based on related literature.

5 Conclusion

In this project, we developed a Markov chain model to analyze recidivism rates of inmates in Florida with regards to their mental health status. We found that effective mental health treatments for inmates can slightly improve overall recidivism rates in the state of Florida and can significantly improve overall mental health status for released inmates in the community.

We chose to study an application of Markov chains as it allowed us to connect our theoretical understanding of eigenvectors and eigenvalues to a concrete, real-world representation. Through working on this project, we learned that there needs to be more research and data collected on the mental health statuses of inmates: how many develop symptoms while incarcerated, how their mental health worsens post-release, and how recidivism factors in. Ultimately, we hope to see more research analyzing the mental health of inmates during and after incarceration in order for inmates to have a healthier transition back into the community and to prevent recidivism.

6 Acknowledgements

We would like to thank Dusty and Caleb for their support throughout the whole project process, as well as our fellow classmates who gave us feedback in their peer reviews. Thanks for a great semester!

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